Pulse Propagation in the Pulmonary Circulation

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Introduction

- Pulmonary circulation regulates an entire cardiac output under low pressure (10-25 mmHg) in normal conditions.
- Abnormally high pressure and significant shape changes in the pressure pulse under hypertensive conditions.
- Wave reflections, originating from junctions and microcirculation, contribute in shaping the pressure pulse.
- Seeking to understand the underlying mechanism of pulmonary pulse propagation under normal and abnormal conditions.
- Investigating the patterns of wave reflections and the parameters influencing their type and nature, such as the pulse wave velocity, due to vascular remodelling.

The Model

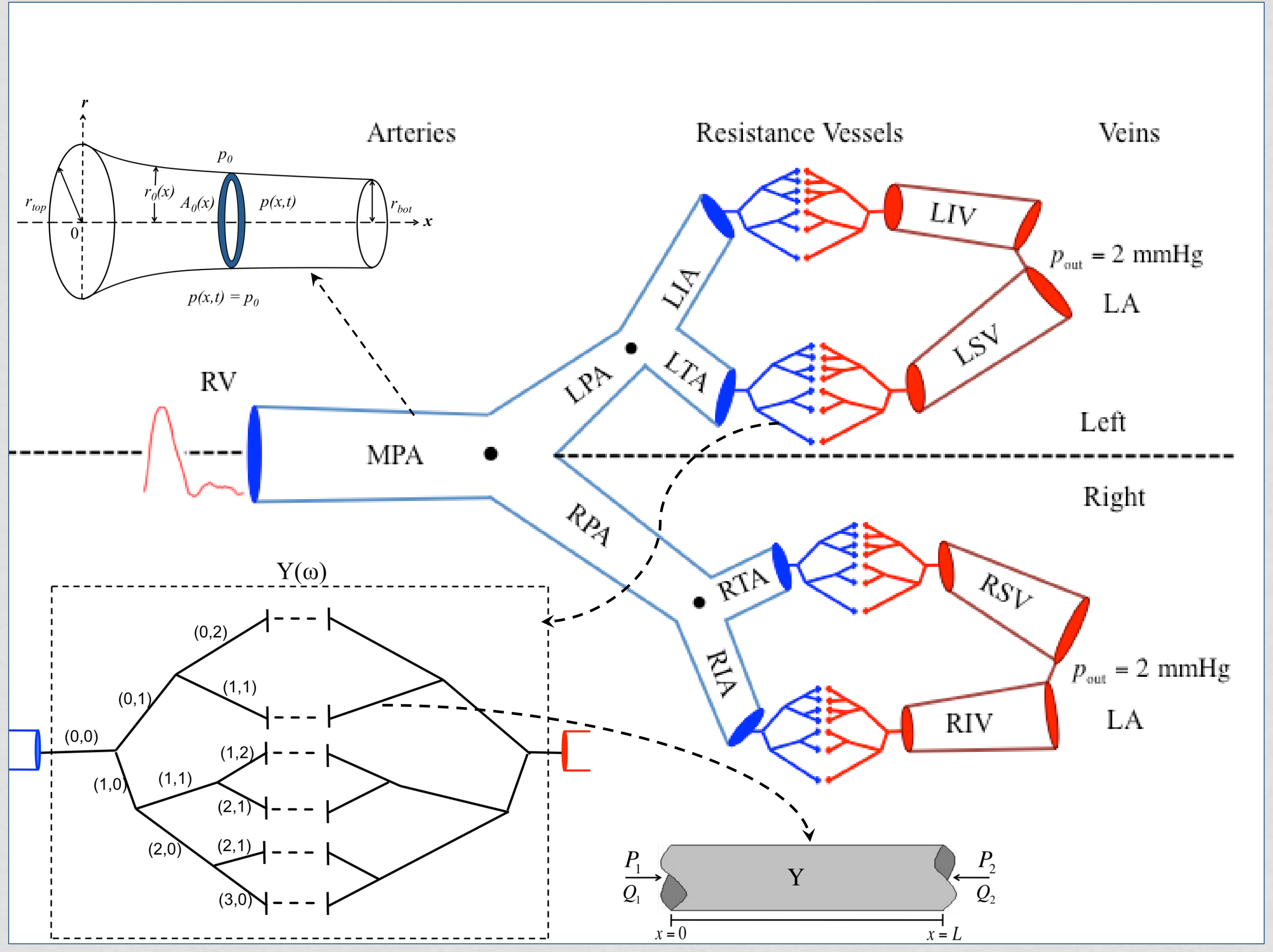


Figure 1. Schematics of basic components and their use in building a complete model of pulmonary system.

- A multi-scale mathematical and computational model.
- Includes both arterial and venous sides of the circulation.
- Patient-specific geometry using magnetic resonance imaging (MRI) for the first three generations of the large pulmonary arteries.
- 'Large arteries' connected with the corresponding 'large veins' via bifurcating and connected structured trees (vascular beds) of 'small vessels' with radii > 50 μm.
- Non-linear fluid dynamical model governing pressure and flow in the large vessels.
- Linearised flow equations for small vessels.

Methods

- **Conservation of Mass and Momentum**

$$\frac{\partial q}{\partial x}+\frac{\partial A}{\partial t}=0 \qquad \frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}\right)+\frac{A}{\rho}\frac{\partial p}{\partial x}=-\frac{2\pi\nu R}{\delta}\frac{q}{A}$$

- **Equation of State**

$$p(x,t)-p_0=\frac{4}{3}\frac{Eh}{r_0}\left(1-\sqrt{\frac{A_0}{A}}\right)$$

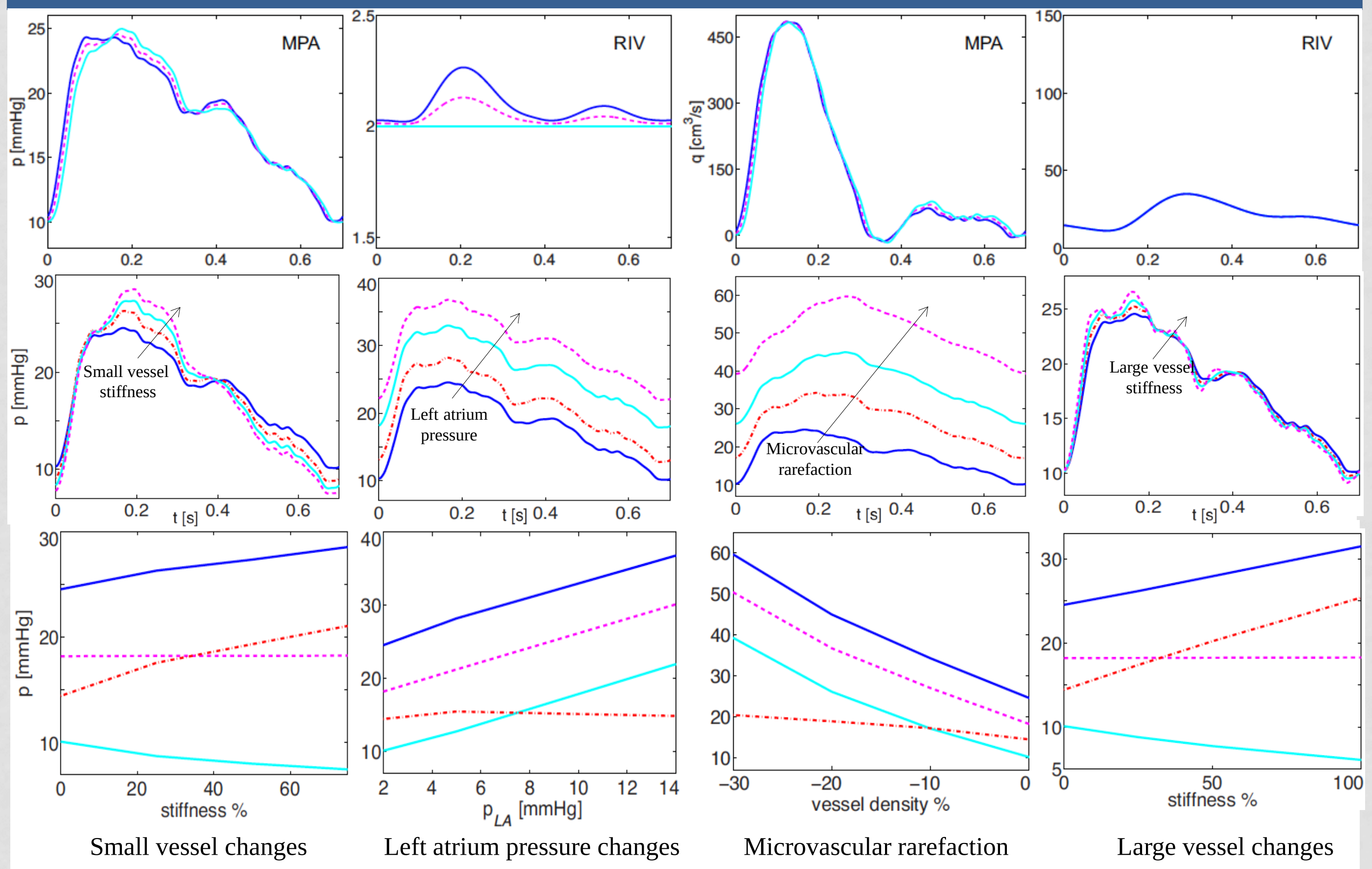
- **Flow and Pressure In the Small Vessels In Frequency Space**

$$Q(x,\omega)=a\cos(\omega x/c)+b\sin(\omega x/c)$$
$$P(x,\omega)=ig_{\omega}^{-1}\left(b\cos(\omega x/c)-a\sin(\omega x/c)\right)$$

- **Admittance Relation Connecting Large Arteries and Veins**

$$\begin{pmatrix}Q_1\\Q_2\end{pmatrix}=\mathbf{Y}\begin{pmatrix}P_1\\P_2\end{pmatrix}$$

Examples of Results



Small vessel changes | Left atrium pressure changes | Microvascular rarefaction | Large vessel changes

Figure 2. 1st Row: Pressure and flow profiles in the MPA and RIV, simulated at proximal, midpoint and distal locations. **2nd Row:** Effects of pulmonary hypertension (PH) on pressure pulse in the MPA.
3rd Row: Effects of PH on mean (magenta), peak (blue), pulse (red) and trough (cyan) pressures in the MPA.

Wave Intensity Analysis (WIA)

- Decomposition of pressure and velocity wavefronts into forward and backward components

$$\mathrm{d}p_{\pm}=\frac{1}{2}(\mathrm{d}p\pm\rho\,c(p)\,\mathrm{d}u), \quad \mathrm{d}u_{\pm}=\frac{1}{2}\left(\mathrm{d}u\pm\frac{\mathrm{d}p}{\rho\,c(p)}\right)$$

- **Wave Intensity**

$$\boxed{\mathrm{W}^{\mathrm{I}}_{\pm}=\frac{\mathrm{d}p_{\pm}}{\mathrm{d}t}\frac{\mathrm{d}u_{\pm}}{\mathrm{d}t}}$$

- **Separated Pressure and Velocity**

$$p_{\pm}=p_0+\int_{t-T}^{t}\mathrm{d}p_{\pm}, \quad u_{\pm}=u_0+\int_{t-T}^{t}\mathrm{d}u_{\pm}$$

Wave Reflections

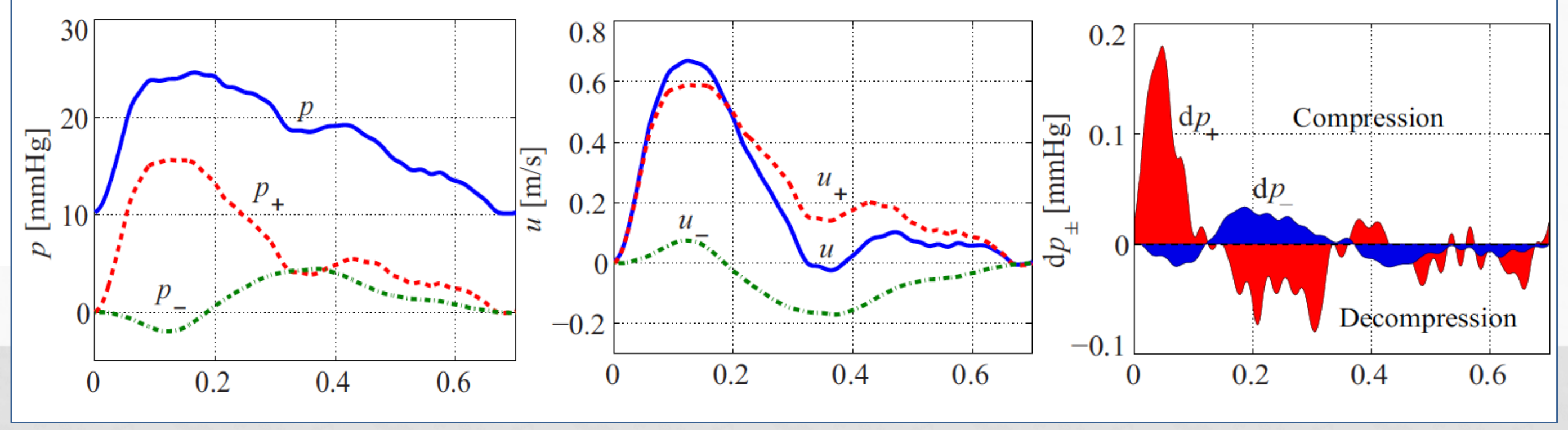


Figure 3. Separated profiles of pressure (left), velocity (center) and pressure wavefronts (right) at the mid-point location in the Main Pulmonary Artery.

- Forward Compression Wave

$$\mathrm{d}p_+ > 0$$

- Forward Decompression Wave

$$\mathrm{d}p_+ < 0$$

- Backward Compression Wave

$$\mathrm{d}p_- > 0$$

- Backward Decompression Wave

$$\mathrm{d}p_- < 0$$

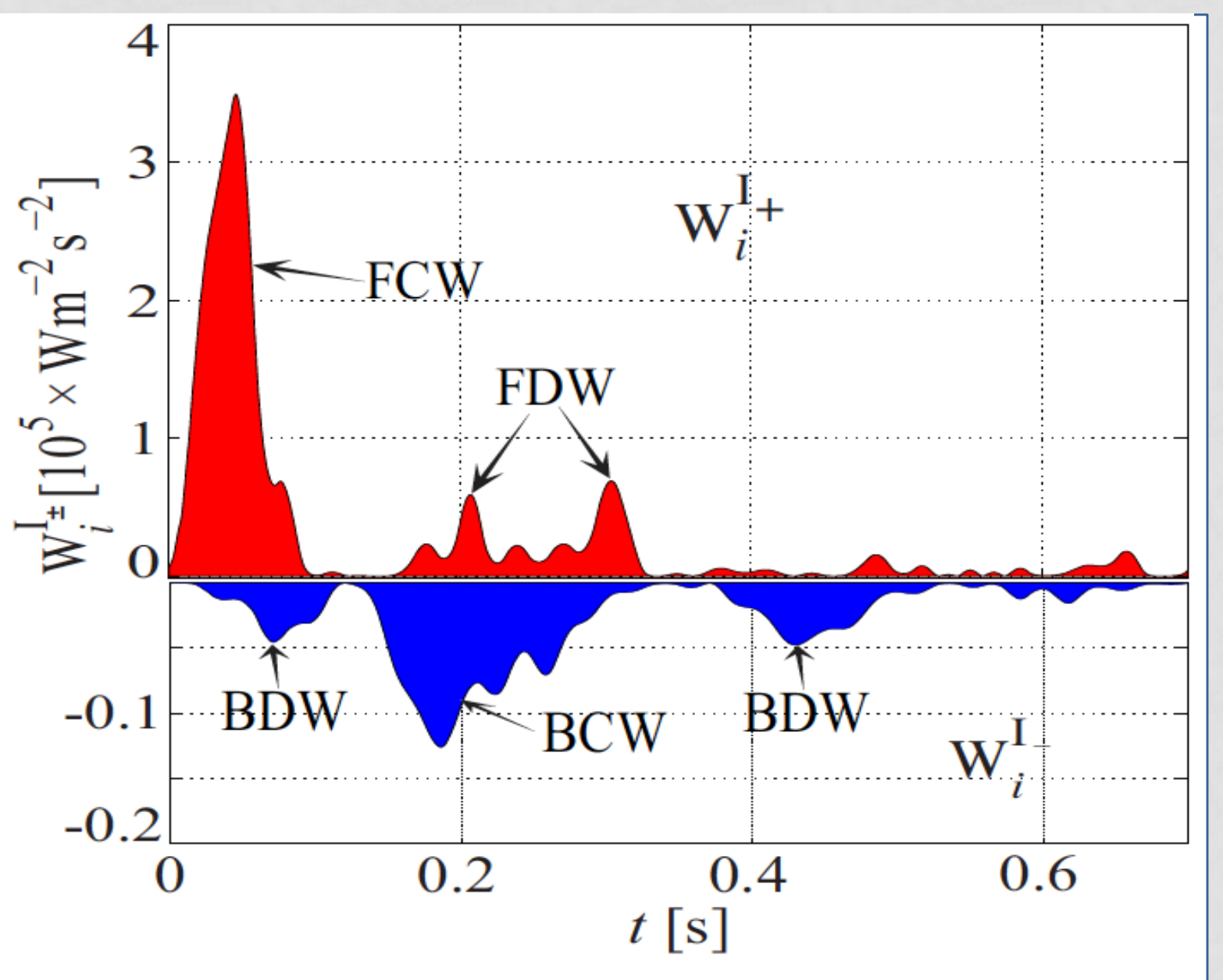


Figure 4. Separated wave intensity profiles in the MPA, identifying the types and direction of local waves.

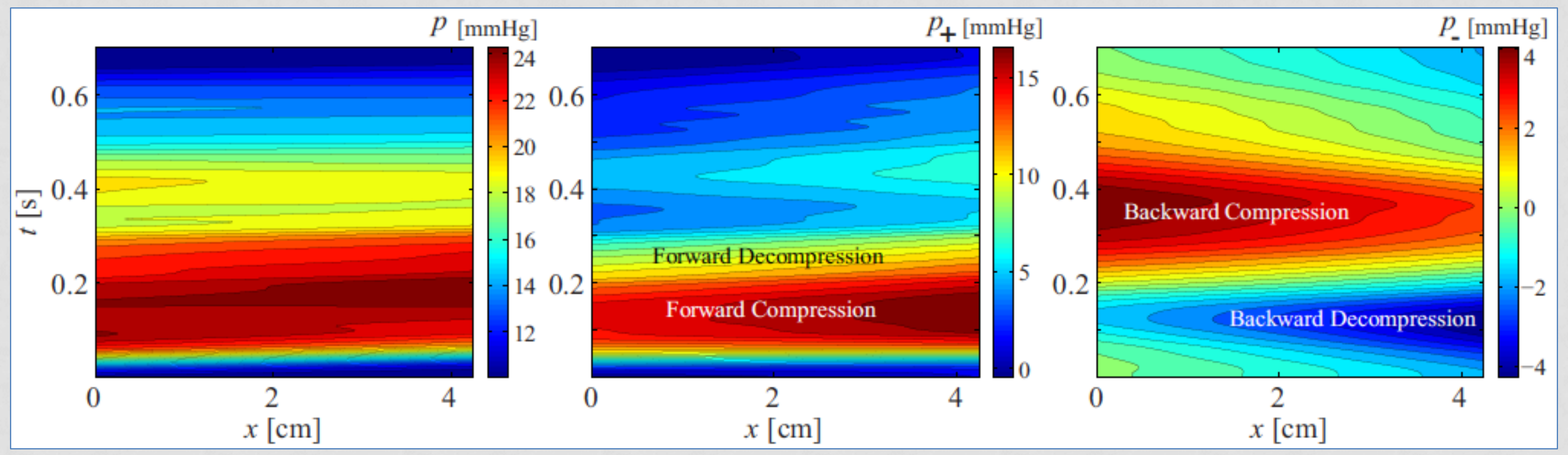


Figure 5. Contours of pressure waveform along the MPA in (x,t) space.

Pulse Wave Velocity (PWV)

- P-U loop Method

$$c_{pu}=\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}u}$$

- Sum of Squares Technique

$$c_{ss}=\frac{1}{\rho}\sqrt{\frac{\sum\mathrm{d}p^2}{\sum\mathrm{d}u^2}}$$

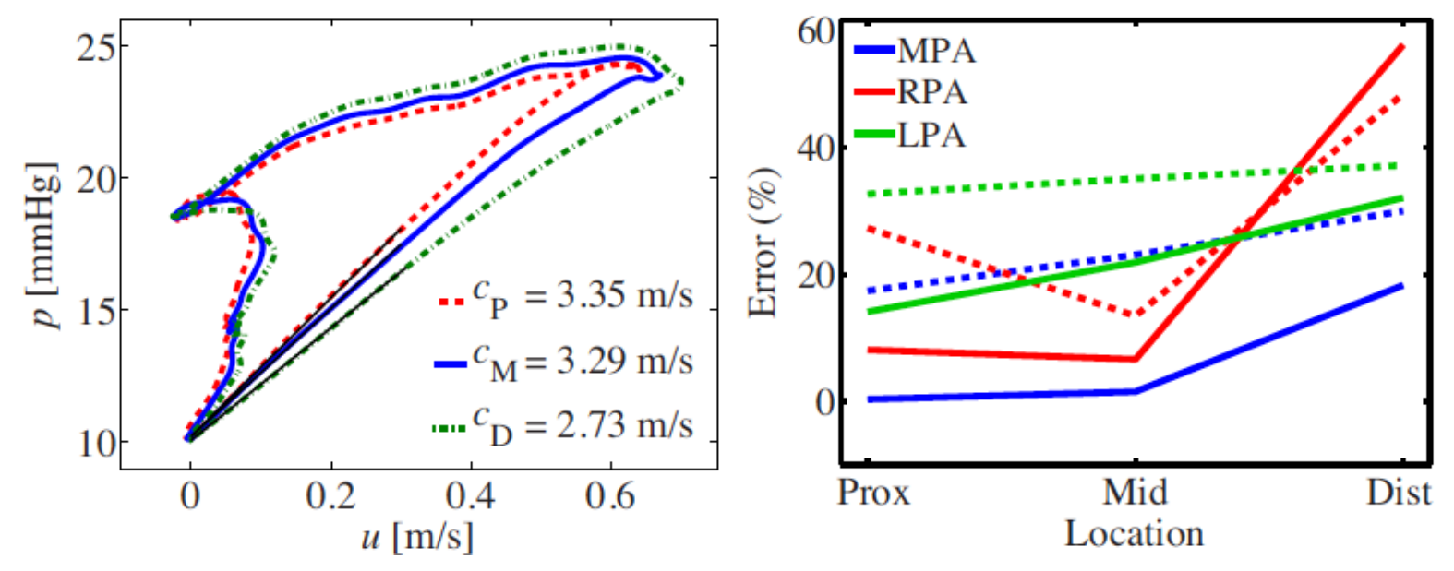


Figure 6. Left: P-U loops in the MPA. Right: Solid lines represent c_{pu} and dashed lines represent c_{ss}. The figure shows the error of PWV estimates about averaged theoretical PWV.

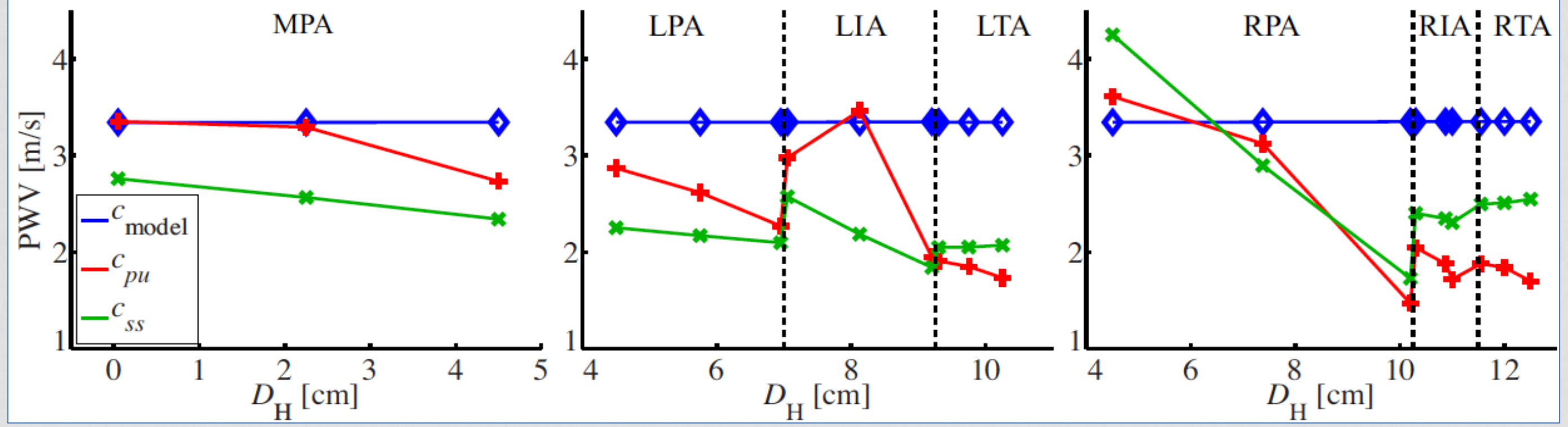


Figure 7. Left: P-U loops in the MPA. Right: Solid lines represent c_{pu} and dashed lines represent c_{ss}. The figure shows the error in PWV estimates about averaged theoretical PWV.

Conclusions

- Increase in peak and pulse pressure during pulmonary hypertension
- WIA demonstrates early systolic negative and late systolic positive wave reflections in the MPA.
- P-U loop is better than Sum of Squares for estimating the PWV at proximal locations.

Contact

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References

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