Uncertainty of 1-D Fluid Models in Pulmonary Hypertension

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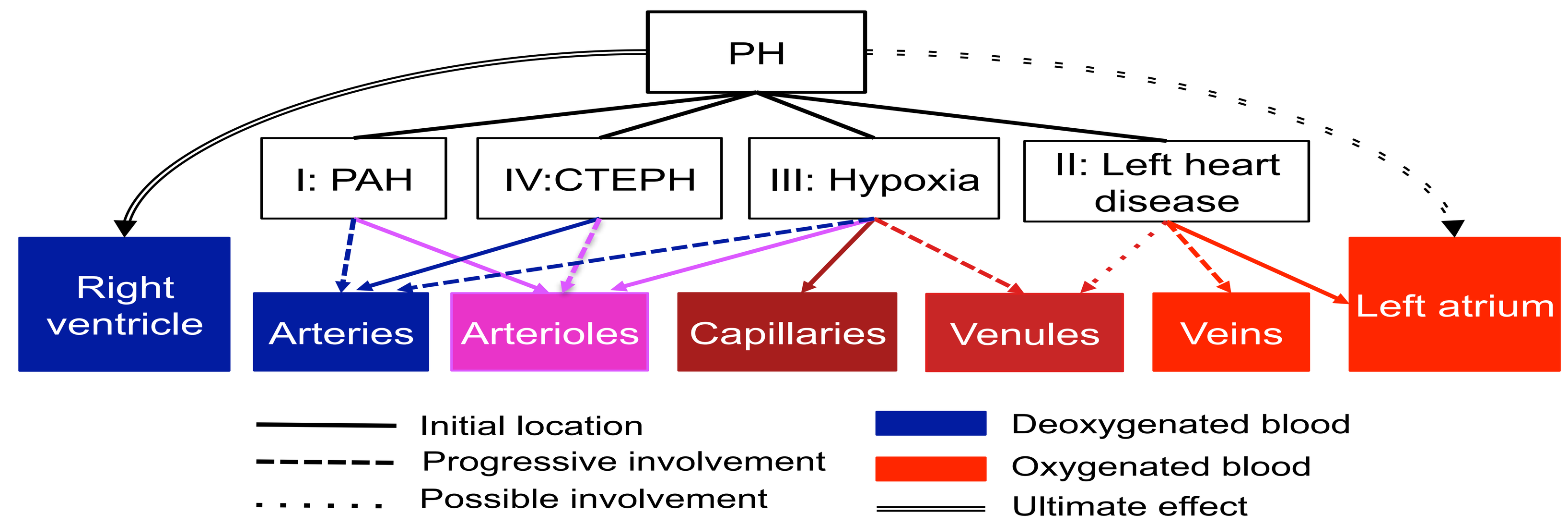
Background

Pulmonary Hypertension (PH) is defined as a mean arterial blood pressure above 25 mmHg in the pulmonary circulation. The condition is classified into 5 groups:

Group I: Pulmonary arterial hypertension (PAH)
Group II: PH due to left heart disease
Group III: PH due to hypoxic (low O_2) lung disease
Group IV: PH due to chronic thromboembolic disease (CTEPH)
Group V: PH due to other miscellaneous causes

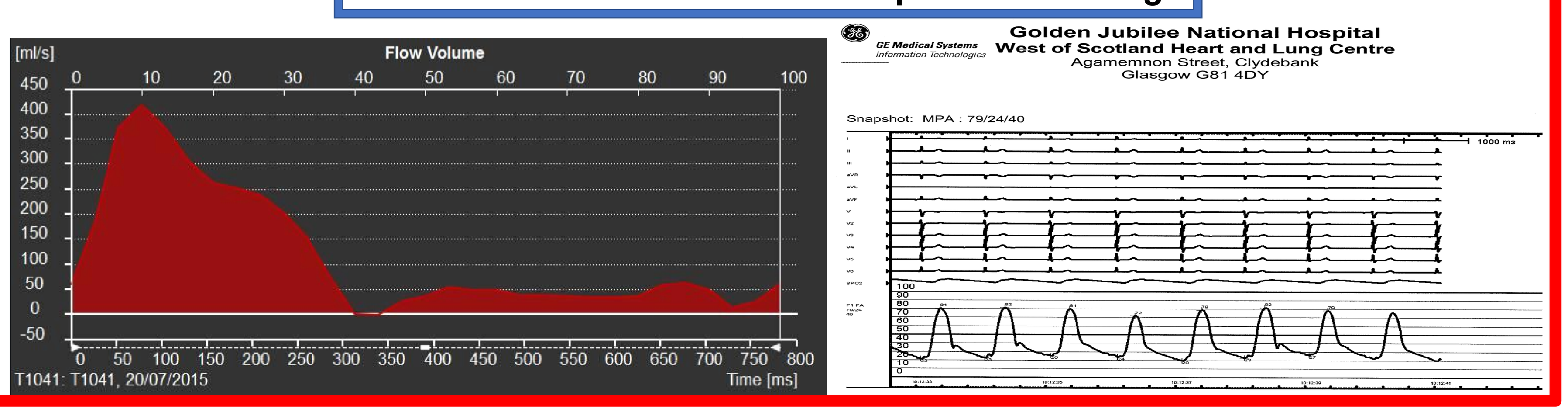
Increased pressure in the pulmonary circulation leads to remodeling of the pulmonary vasculature, causing increased arteriolar stiffness and thus increased pressure in large pulmonary arteries.



Data Analysis

The gold standard for medical testing in PAH is right heart catheterization, during which a catheter is inserted into either a vein in the groin or neck region and is then moved to the heart. The catheter is moved from the right side of the heart, to the main pulmonary artery, and then to any other arteries, recording continuous pressure wave forms. Patients are then put into an MRI scan, during which continuous blood flow and cardiac output are measured.

Continuous MRI flow and invasive pressure readings



Vascular Geometry and Uncertainty

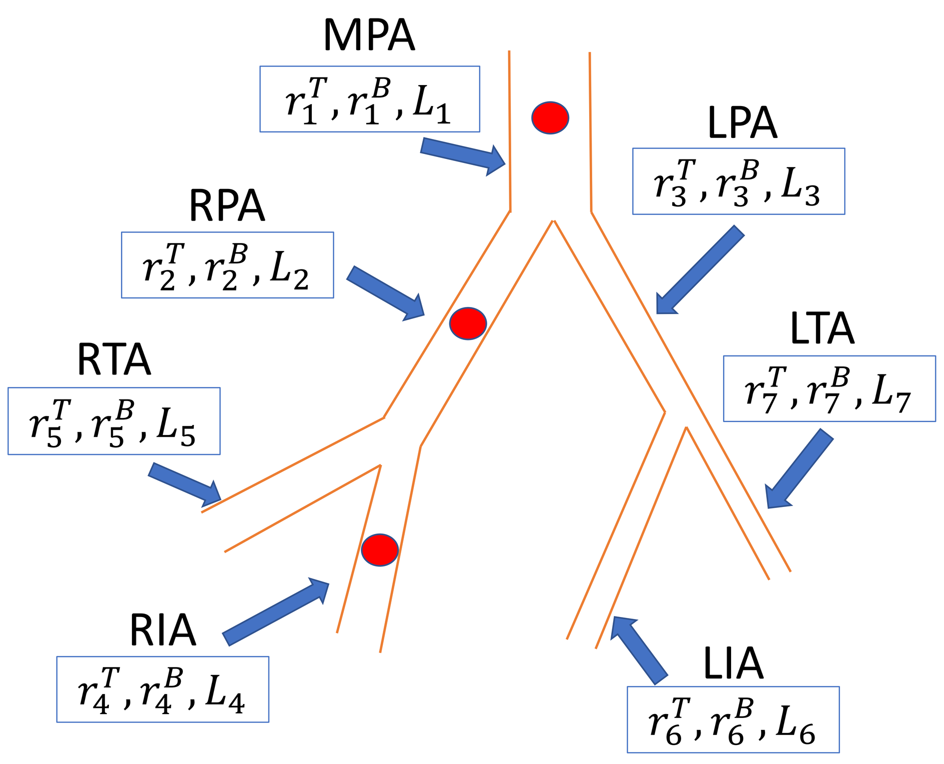
- Obtained via a mixture of MRI and CT imaging
- Non-invasive measurements have inherent measurement error, thus leaving
- To study the impact of geometry, we define $\boldsymbol{\theta}=\{r_i^T, L_i\}_{i=1}^7$ as the measured geometry values via MRI, and take 1000 random draws from a normal distribution, i.e.

$$\tilde{r}^T_{ij} \sim N(r_i^T, \psi), \; j=1,2,\dots 1{,}000$$
$$\tilde{L}_{ij} \sim N(L_i, \psi), \quad j=1,2,\dots 1{,}000$$

- Using the geometry $\widetilde{\theta_j}=\{\tilde{r}^T_{ij}, \tilde{L}_{ij}\}$, we simulate pressure and flow in the network

Measured radii and length from MRI

Vessel	r_i^T [mm]	r_i^B [mm]	L_i [mm]
MPA (i=1)	16.63	15.46	60.61
RPA (i=2)	12.37	12.13	65.95
LPA (i=3)	11.4	7.81	43.04
RIA (i=4)	5.03	5.03	26.12
RTA (i=5)	11.08	11.08	32.37
LIA (i=6)	5.62	5.62	10.55
LTA (i=7)	5.53	5.53	20.6



Acknowledgements

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Mathematical Model

1. Conservation of Mass and Momentum

$$\frac{\partial q}{\partial x}+\frac{\partial A}{\partial t}=0, \qquad \frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}\right)+\frac{A}{\rho}\frac{\partial P}{\partial x}=\frac{2\pi\nu R}{\delta}\frac{q}{A}$$

2. Equation of State (Pressure Area Relation)

$$P(x,t)=P_0+\frac{4}{3}\frac{Eh}{r_0}\left(1-\sqrt{\frac{A_0}{A}}\right), \qquad \frac{Eh}{r_0}\approx k_1e^{k_2r_0}+k_3$$

3. Junction Condition

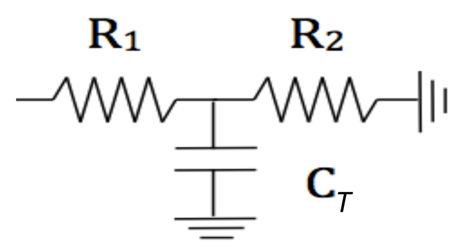
$$p_p(L,t)=p_{d_i}(0,t)$$
$$q_p(L,t)=\sum_{i=1}^{2}q_{d_i}(0,t)$$

4. Outflow Boundary Conditions

WindKessel

$$\theta_{WK}=\{R_1,R_2,C_T\}$$

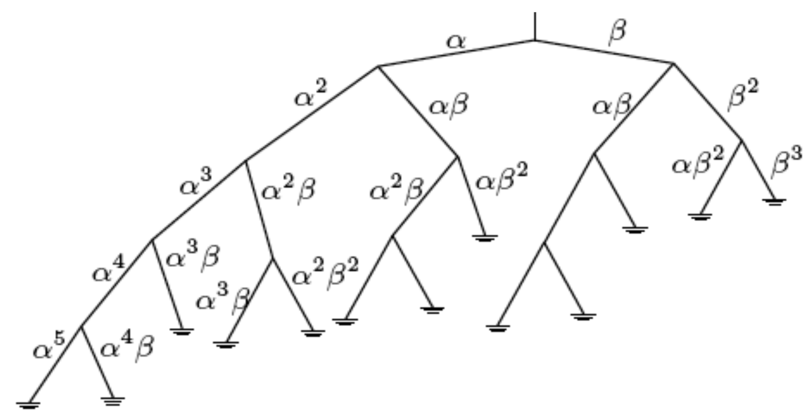
$$\frac{dp}{dt}=R_1\frac{dq}{dt}-\frac{p}{R_2C_T}+\frac{R_1+R_2}{R_2}\frac{q}{C_T}$$



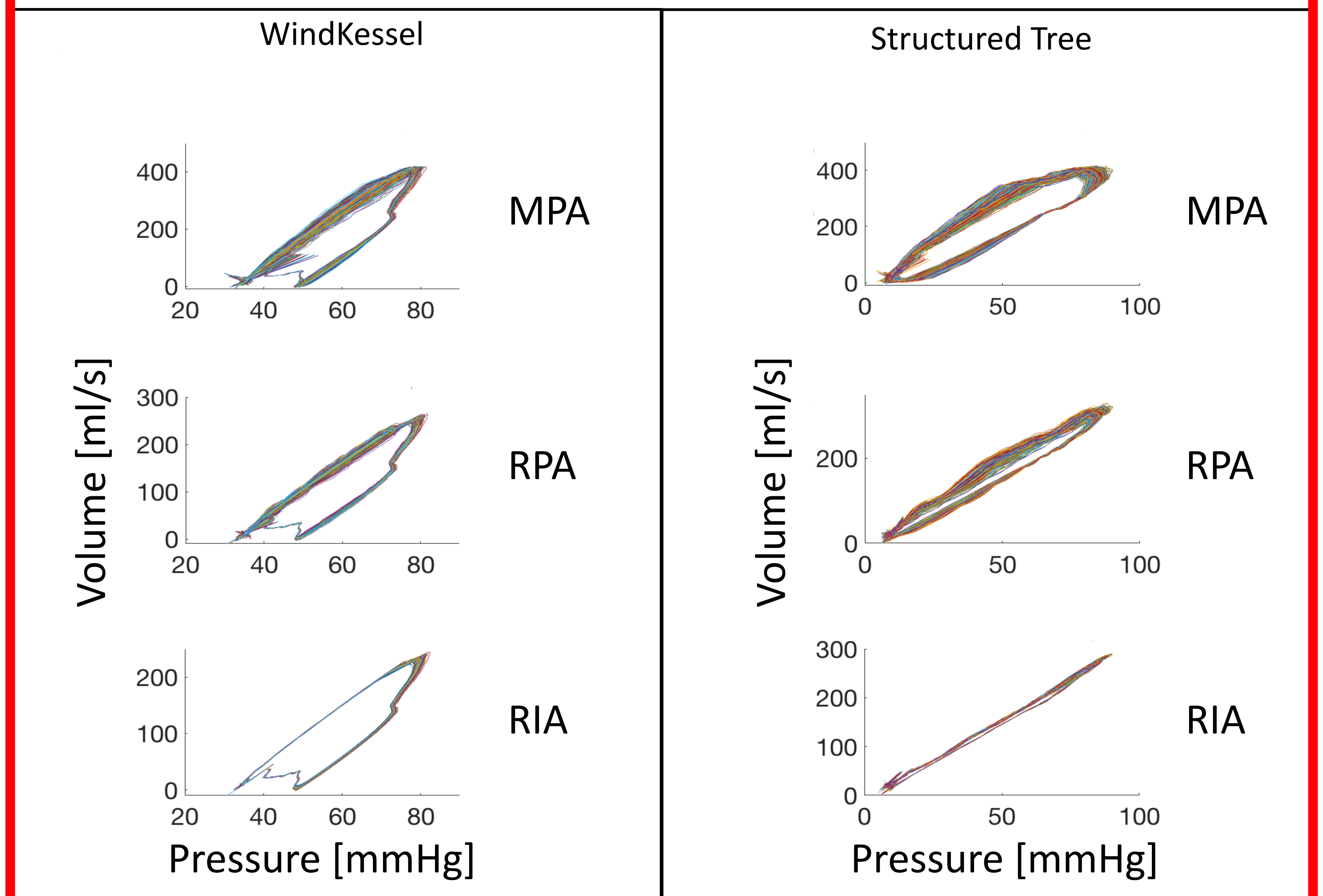
Structured Tree

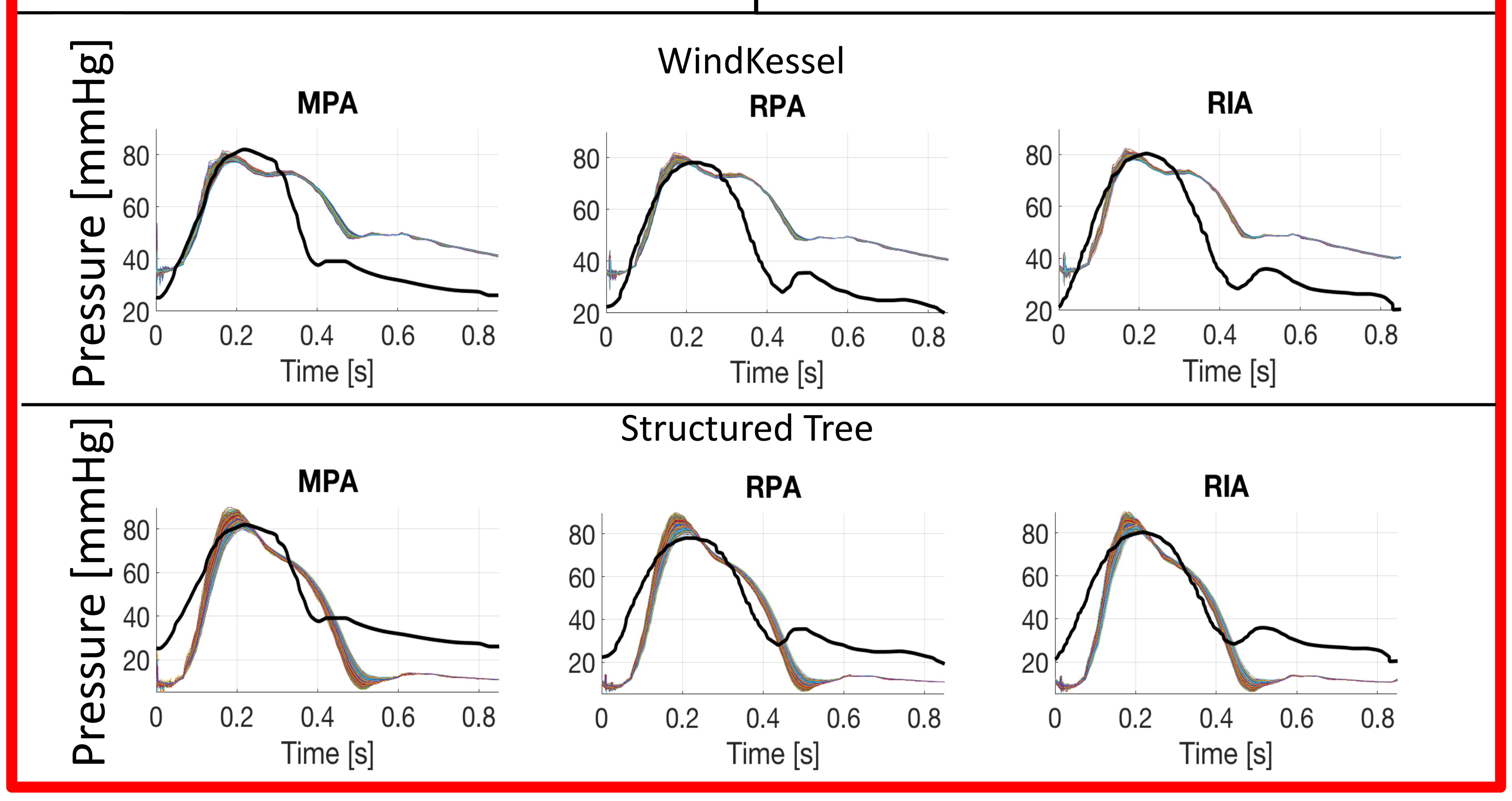
$$\theta_{ST}=\{\alpha,\beta,L_{rr}\}$$

$$\alpha=\left(1+\gamma^{\xi/2}\right)^{-1/\xi},\ \beta=\alpha\sqrt{\gamma},\ L_{rr}=\frac{L_i}{r_i}$$



Results





Conclusions

- Network geometry plays a critical role in prediction of flow and pressure
- Insight into vessel uncertainty can help identify variability in waveforms
- Variability in geometry creates more pronounced effects in structured tree than in Windkessel